

Dynamic Auctions with Budget-Constrained Bidders: Evidence from the Online Advertising Market





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Motivation

- Auctions are used in many real-world contexts
 - Price discovery and buyer competition
- Led to extensive theoretical and empirical research
 - focusing on bidders facing only one auction
- However, firms and consumers often face auctions sequentially
- Dynamics become important if financially constrained
 - Little empirical attention
- Ex: Financial markets, energy markets, eBay, **online ad market**

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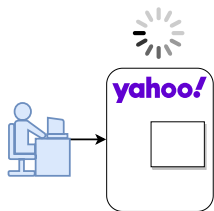


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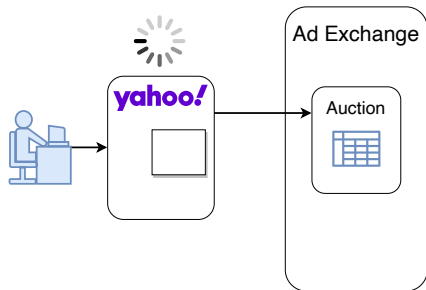
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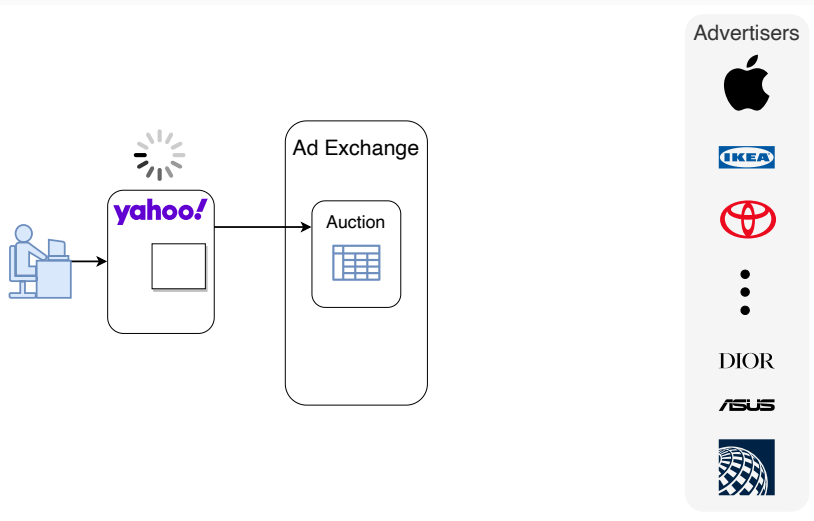
Online Advertising Auctions (Real-Time Bidding)



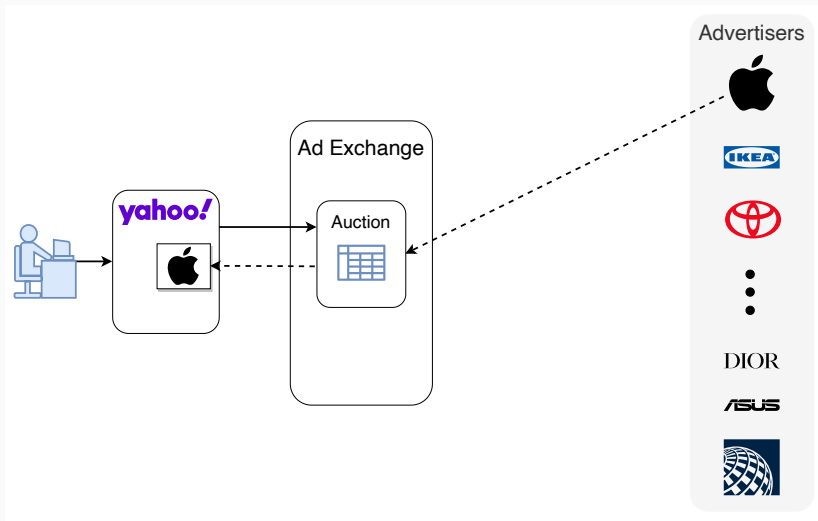
Online Advertising Auctions (Real-Time Bidding)



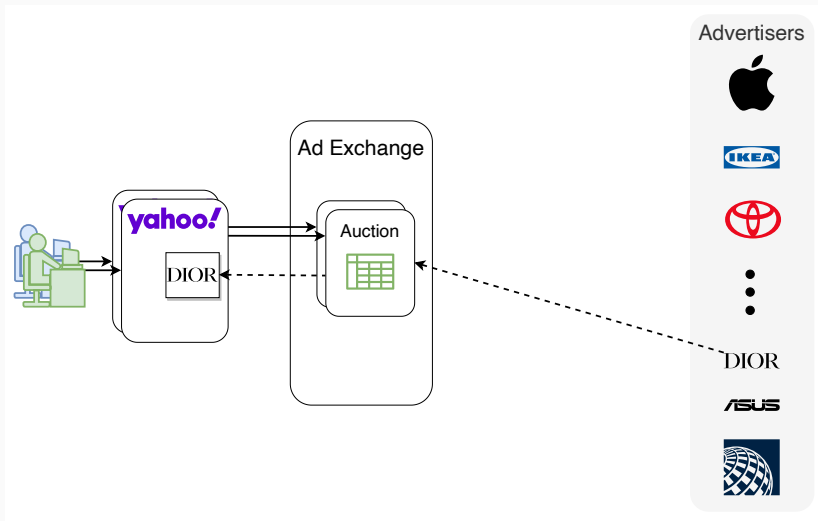
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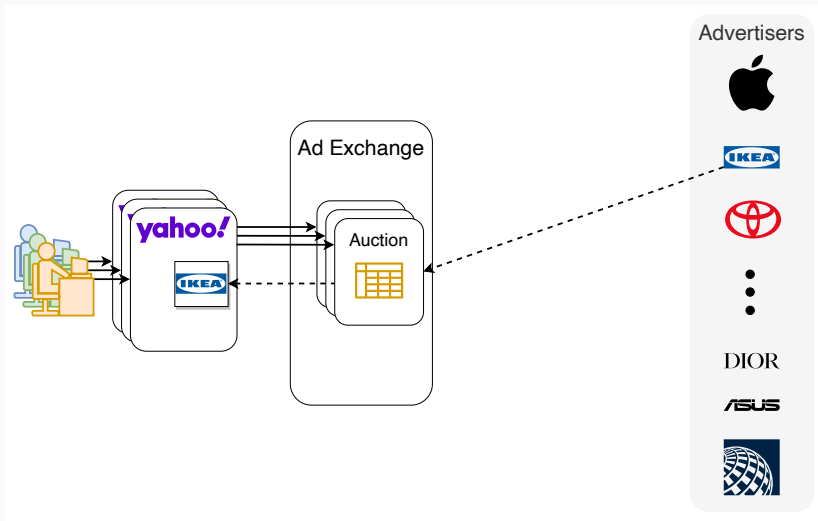
Online Advertising Auctions (Real-Time Bidding)



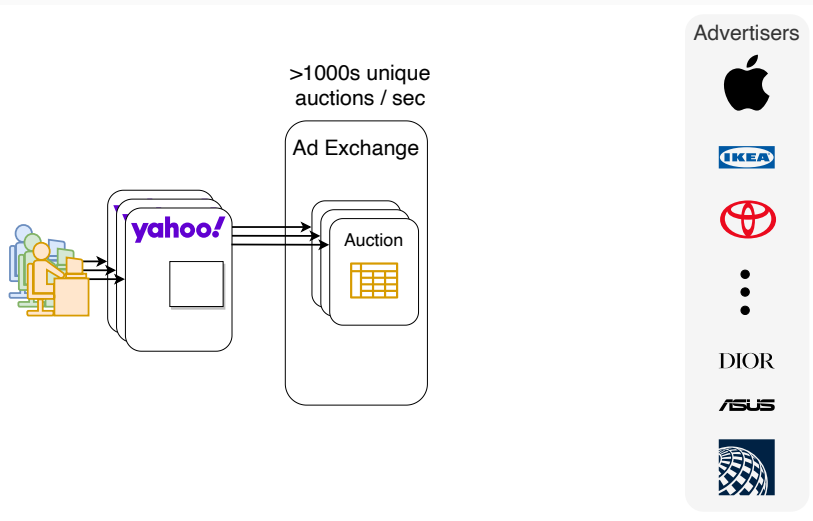
Online Advertising Auctions (Real-Time Bidding)



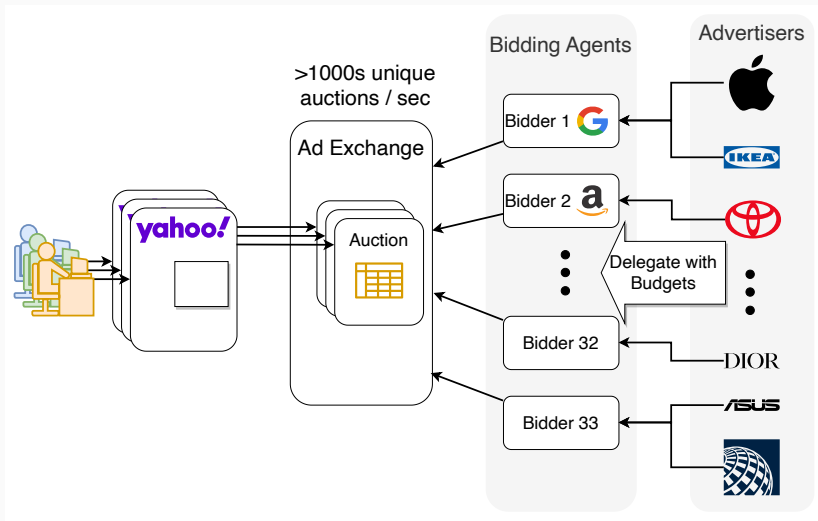
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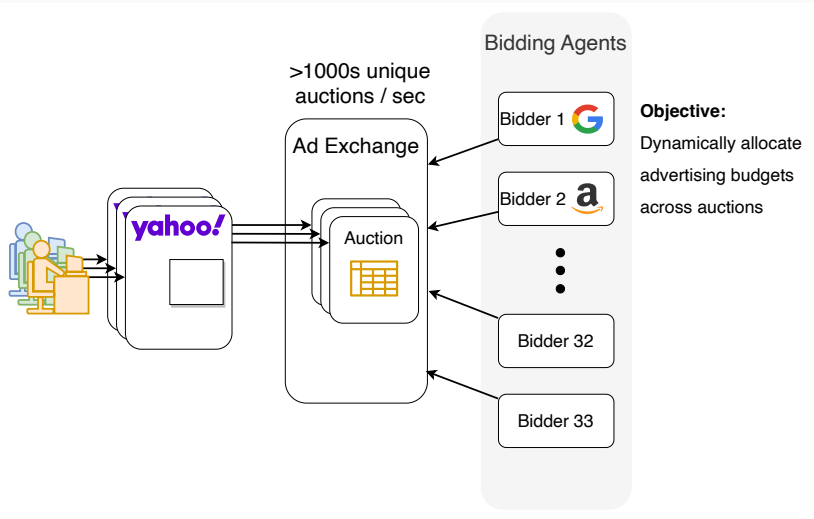
Online Advertising Auctions (Real-Time Bidding)



Online Advertising Auctions (Real-Time Bidding)



Online Advertising Auctions (Real-Time Bidding)



Research Questions

Dynamic auctions with budget-constrained bidders

- How do bidders dynamically compete against each other?
- How does the auction format affect the dynamic competition?

This paper

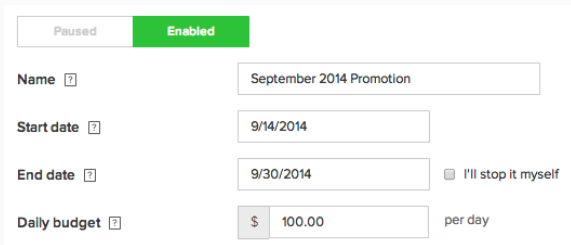
- First empirical analysis of dynamic auctions with budget constraints
 - using a novel proprietary dataset of online ad auctions
- Novel structural framework of dynamic auctions with budget-constrained bidders
- Quantify how dynamic constraints shape bidders' strategic behavior
 - Substantial impact on participation and bid decisions
 - Heterogeneity in budgets \Rightarrow Heterogeneity in dynamic behavior
- Counterfactual simulation to analyze mechanism design
 - Ongoing debate: First-price auction vs Second-price auction
 - First-price auction better for bidders with smaller budgets

Roadmap

1. Background
2. Descriptive Evidence
3. Structural Model
4. Estimation / Results
5. Counterfactuals

Background: Budgets and Bidding Agents

- Advertisers set up ad campaigns with bidding agents
- Campaign settings: Goal, target audience, length, and **budget**
- Generally enforced through **daily budget constraints**



The image shows a screenshot of an ad campaign configuration interface. At the top, there are two buttons: "Paused" and "Enabled", with "Enabled" being highlighted in green. Below this, there are four rows of input fields:

- Name**: A text input field containing "September 2014 Promotion".
- Start date**: A date input field containing "9/14/2014".
- End date**: A date input field containing "9/30/2014", followed by a checkbox labeled "I'll stop it myself" which is currently unchecked.
- Daily budget**: A currency input field showing "\$ 100.00" and the text "per day" to its right.

- Bidding agents strategize while adhering to daily budgets

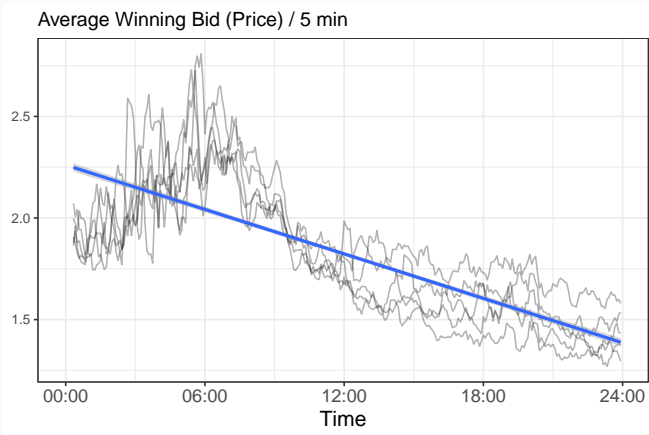
- Bid-level data on ad auctions hosted on Yahoo ad exchange
 - Current industry standard: First-price auctions
- One week data from the second quarter of 2021
- 16 websites owned by Yahoo (Finance, News, etc)
- 33 bidding agents and 71,011 advertisers.

Roadmap

1. Background
2. Descriptive Evidence
 - Empirical patterns consistent with intertemporal budget constraints
3. Structural Model
4. Estimation / Results
5. Counterfactuals

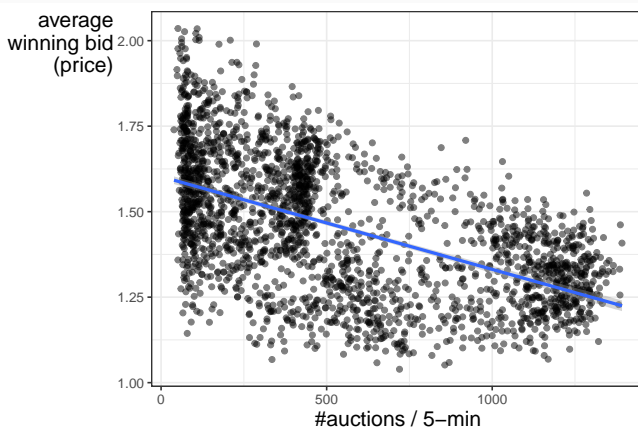
Observation 1: Declining Price

- Entry rate and bid level decline within each day
⇒ declining average price (winning bid)



Observation 2: Bidders' respond to freq. of auctions

- When the freq. of auctions is higher, bidders enter auctions at a lower rate and submit lower bids
- 10% \uparrow in freq. of auctions \implies 1.3% \downarrow in price
 - while controlling for auction characteristics and hour FE



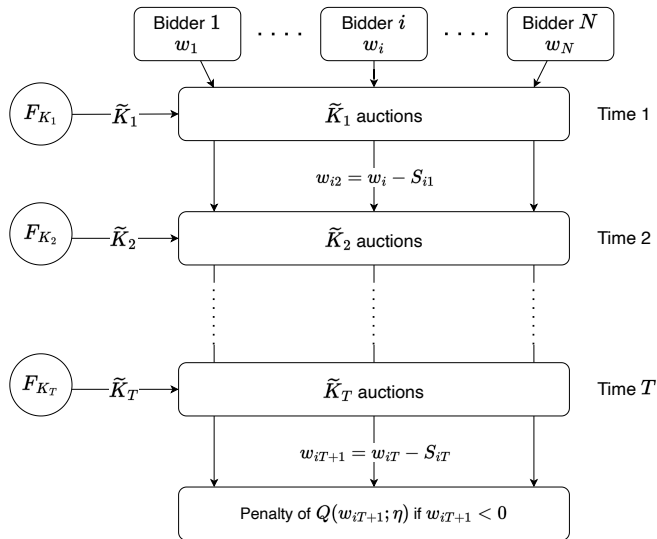
Review of Stylized Facts

- Consistent patterns connected to budget constraints:
 1. Declining average price within each day
 2. Inverse relationship between the freq. of auctions and competition
- Suggests classical implications may not apply
- Need a new model to conduct analysis

Roadmap

1. Background
2. Descriptive Evidence
3. **Structural Model**
 - Recover bidders' budgets & valuations for impressions
4. Estimation / Results
5. Counterfactuals

Dynamic Auctions with Budget-Constrained Bidders



In our application,

$N = 33$

$T = 24$

Model Implications

- Analyze the best-response problem under an oblivious equilibrium concept (Weintraub et al., 2008)
- Auctions are strategically linked together unlike standard models
- Intertemporal budget constraints introduce a dynamic tradeoff
- Entry and bid strategies depend on
 - frequency of auctions K_t
 - remaining budget w_{it}
 - current and future competitiveness

Roadmap

1. Background
2. Descriptive Evidence
3. Structural Model
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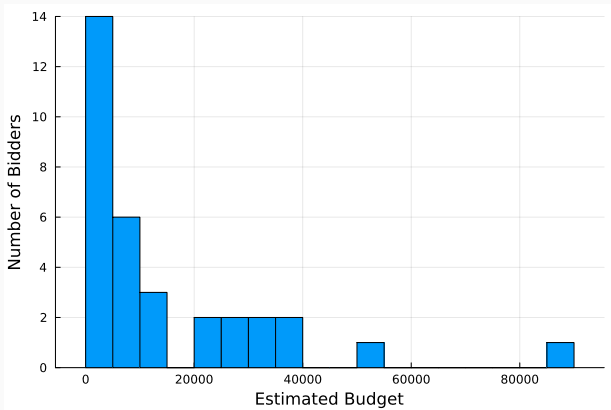
Using our novel dataset from Yahoo, we take a two-step approach to avoid equilibrium computation¹:

1. Estimate each bidder's belief over other players' behavior
2. MLE to estimate the model primitives using the structural model
 - Inner loop: solving the best-response problem via backward induction

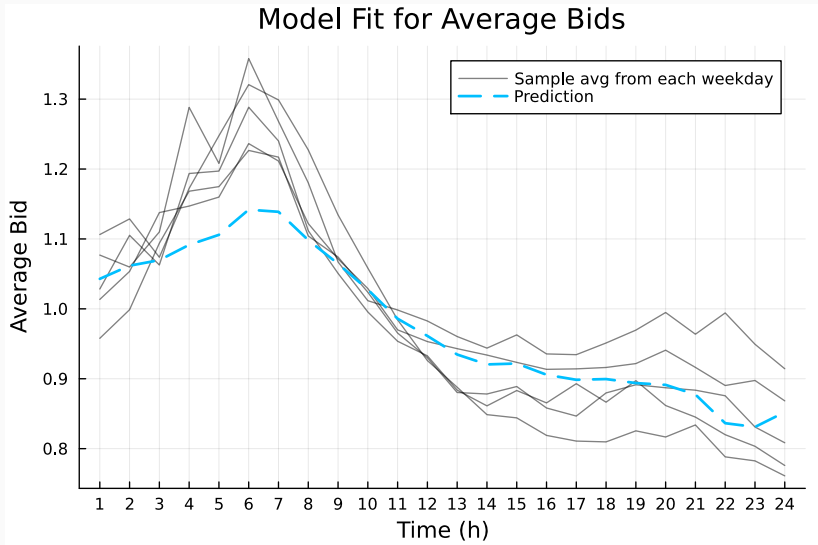
¹Bajari et al. (2007); Aguirregabiria and Mira (2007)

Estimation Results

- Statistically significant evidence that budget constraints matter
 - Spending exceeds budget approximately 26% of the time, but amounts to around 8% of budget plot
- Significant heterogeneity in $(w_i)_{i=1}^N$
 - \Rightarrow heterogeneity in entry & bid behavior plot



Model Fit



Roadmap

1. Background
2. Descriptive Evidence
3. Structural Model
4. Estimation / Results
5. Counterfactuals

First-price vs Second-price auction

- CF to generate insights for mechanism design
- We use our equilibrium solver to compare FPA and SPA
 - SPA is no longer strategy-proof
 - No prior welfare comparison
- Motivated by an institutional shift that happened after 2018
 - Sellers and buyers demanded more transparency from ad exchanges
- Conventional analysis:
 - $FPA = SPA$ in revenue and welfare considerations
- In dynamic auctions with budget-constrained bidders, we find significant difference in welfare outcomes

FPA vs SPA: Dynamics

- Price variance is lower in FPA than in SPA
 - Similar to the standard case (Krishna, 2009)
- Bidders with smaller budgets are more aggressive under FPA
- Larger bidders are pressured to spend rapidly
 - In later periods, they become tighter constrained
- Smaller bidders capitalize on the reduction in competition
 - Better off under FPA than SPA
- Total welfare and revenue are both marginally higher under FPA

FPA vs SPA: Implications

- Policy
 - FPA $>$ SPA for smaller firms
 - FPA encourages smaller bidders to be more competitive against larger bidders
- Dynamic mechanism design with financial constraints
 - Price volatility determines dynamic competition
 - Not specific to daily budget constraints

Conclusion

- Propose a novel structural framework of dynamic auctions with budget-constrained bidders
- Empirically analyze how budget constraints shape dynamic competition in the online advertising market
 - First empirical analysis
- Find significant impact from budget constraints which is heterogeneous across bidders
- First-price auction better at encouraging smaller bidders to be more competitive
 - driven by the difference in price volatility

Thank you!

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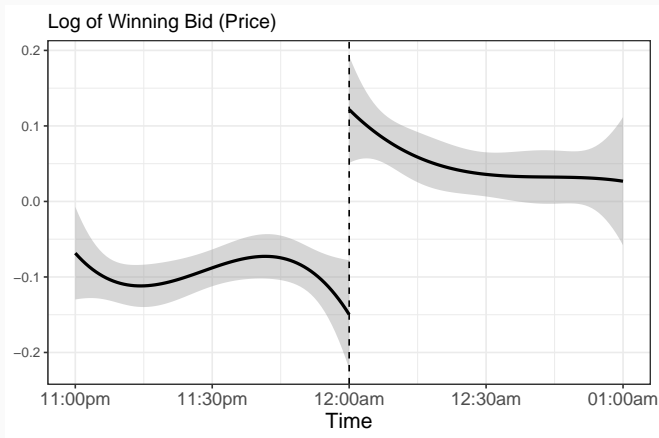
Summary Statistics

variable	n	mean	std	min	median	max
Bid	8,856,603	1.000	1.682	0.061	0.577	369.070
# entrants	1,229,300	7.205	4.732	1.000	7.000	25.000
Win bid	1,229,300	2.294	3.441	0.061	1.182	369.070

- Note: avg bid normalized to 1 in sample (confidentiality)

Observation 2: Price jumps when budgets are renewed

- There is a consistent jump at midnight
- Reg. Discontinuity: Price jumps by $\sim 40\%$ on average
 - while controlling a rich set of auction characteristics



Stage Game

- State variables: $K_t, (w_{it})_{i=1}^N$
- K_t first-price auctions
 - Private entry cost: $C_{ikt} \stackrel{iid}{\sim} F_C$
 - Private valuation: $X_{ikt} \stackrel{iid}{\sim} F_X$
- Strategies chosen before C_{ikt} and X_{ikt} are realized
 - Entry threshold strategy: $\bar{c}_{it} \geq 0$
 - Bid strategy: $x \in \mathbb{R} \mapsto b_{it}(x) \in \mathbb{R}$
 - If $C_{ikt} \leq \bar{c}_{it}$, submits $b_{it}(X_{ikt})$
- Stage payoffs

$$\sum_{\text{auctions won}} (X_{ikt} - b_{it}(X_{ikt})) - \sum_{\text{auctions entered}} C_{ikt}$$

- Payment

$$S_{it} = \sum_{\text{auctions won}} b_{it}(X_{ikt})$$

Solution Concept

- Bidders have little info on their rivals' spending
- Large-market equilibrium: Each bidder tracks only (K_t, w_{it})
 - similar to oblivious equilibrium (Weintraub et al., 2008)
- Eqb. object: Prob. of winning, CDF of the highest competing bid

$$\begin{aligned}\Psi_t(b | K_t) &= \Pr \left(\max_{j \neq i} B_{jkt} \leq b \mid K_t \right) \\ &= E_{(w_{jt})_{j \neq i}} \left[\underbrace{\Pr \left(\max_{j \neq i} B_{jkt} \leq b \mid K_t, (w_{jt})_{j \neq i} \right)}_{\text{Determined by } (\bar{c}_t(K_t, w_{jt}), b_t(\cdot | K_t, w_{jt}))} \right]\end{aligned}$$

Bidding Problem

- Take $\Pr(\text{win}) = \Psi_t(\cdot | K_t)$ as given
- Best-response entry and bid strategies solved via backward induction

$$V_t(K_t, w_{it}) = \max_{\bar{c}, b(\cdot)} \underbrace{K_t F_C(\bar{c})}_{\text{Expected \#Entry}} \left(\underbrace{E[\Psi_t(b(X) | K_t)(X - b(X))]}_{\text{Expected Surplus}} - \underbrace{E[C | C \leq \bar{c}]}_{\text{Expected Entry Cost}} \right) + \underbrace{E[EV_{t+1}(w_{it} - S_{it}) | b(\cdot), \bar{c}]}_{\text{Continuation Value}}$$

where

$$EV_{t+1}(w) = E_{K_{t+1}}[V_{t+1}(K_{t+1}, w)]$$

$$EV_{T+1}(w) = -Q(w; \eta)$$

- If no constraint ($\eta = 0$), static auction with entry
 - Li and Zheng (2009)

Model Primitives

Using our novel dataset of online ad auctions from Yahoo, we estimate:

1. F_{K_t} : Time-variant distribution of #auctions
2. F_C : Distribution of entry costs
3. F_X : Distribution of valuations for ad opportunities
4. η : Parameter in penalty $Q(w_{iT+1}; \eta) = \eta w_{iT+1}^2 \mathbb{1}_{\{w_{iT+1} < 0\}}$
5. $(w_i)_{i=1}^N$: Bidders' budgets

identification

Why Daily Budgets?

- Advertisers impose them on bidding agents
- They may want them for a few reasons
 1. Preventing overspending from an error
 2. Ensuring consistent exposure
 3. Convenience for accounting/billing
- Our structural model could be used to assess a counterfactual change

◀ background

◀ figure

◀ review

Why Declining Price?

- Similar theoretical findings in sequential auctions with unit-demand bidders with independent valuations²
- Large heterogeneity in ad opportunities
- When bidders find a consumer that matches well with their ad, they have delay costs.
 1. they may face worse objects later
 2. they are not guaranteed to win later

◀ figure

◀ review

²Bernhardt and Scoones (1994); Engelbrecht-Wiggans (1994); Gale and Hausch (1994)

First-Order Conditions

- Entry threshold FOC

$$\bar{c} = \underbrace{E[\Psi_t(b(X) | K_t)(X - b(X))]}_{\text{Static Threshold}} + \underbrace{\frac{1}{K_t f_C(t)} \frac{\partial}{\partial \bar{c}} E[EV_{t+1}(w_{it} - S_{it}) | b(\cdot), \bar{c}]}_{\text{Dynamic Tradeoff}}$$

First-Order Conditions

- Entry threshold FOC

$$\bar{c} = \underbrace{E[\Psi_t(b(X) | K_t)(X - b(X))]}_{\text{Static Threshold}} + \underbrace{\frac{1}{K_t f_C(t)} \frac{\partial}{\partial \bar{c}} E[EV_{t+1}(w_{it} - S_{it}) | b(\cdot), \bar{c}]}_{\text{Dynamic Tradeoff}}$$

- Bid strategy FOC

$$E \left[\underbrace{\left(X - \frac{\Psi_t(b(X) | K_t)}{\Psi'_t(b(X) | K_t)} - b(X) \right)}_{\text{Static FOC}} \Psi'_t(b(X) | K_t) \nabla_\gamma b(X) \right] + \underbrace{\frac{1}{K_t F_C(\bar{c})} \nabla_\gamma E[EV_{t+1}(w_{it} - S_{it}) | b(\cdot), \bar{c}]}_{\text{Dynamic Tradeoff}} = 0$$

Identification of η and $(w_i)_{i=1}^N$

- Key assumption: $X_{ikt} \perp (K_t, (S_{is})_{s=1}^{t-1})$
- Intuition: capture the observed correlation by the dynamic tradeoffs
- Plausible since valuations are computed based on click/sale probability and value from such events.

moment condition

◀ back

Moment Condition

- For the correct parameters θ , we have

$$B_{ikt} = b_t \left(X_{ikt} \mid K_t, w_i - \sum_{s=1}^{t-1} S_{is}; \theta \right)$$

- $X_{ikt} \perp (K_t, (S_{is})_{s=1}^{t-1})$ implies

$$E \left[b_t^{-1} \left(B_{ikt} \mid K_t, w_i - \sum_{s=1}^{t-1} S_{is}; \theta \right) - E[X_{ikt} \mid \theta] \mid K_t, (S_{is})_{s=1}^{t-1} \right] = 0$$

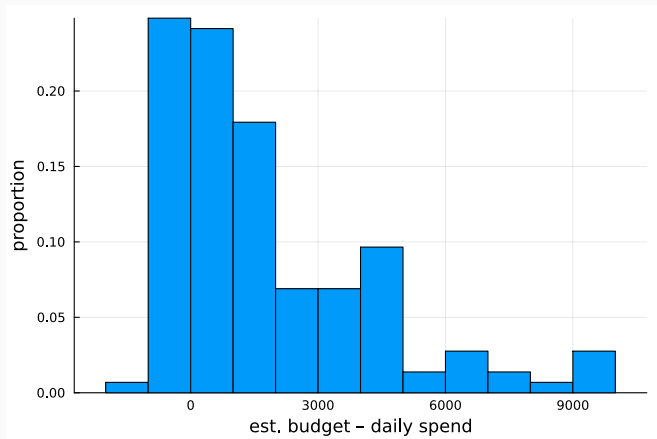
Estimated structural parameters

Parameters	Estimate	SE
μ_C	-11.3776	0.0091
σ_C	7.2533	0.0062
μ_X	0.9046	0.0007
σ_X	1.0950	0.0006
η	0.6457	0.0084

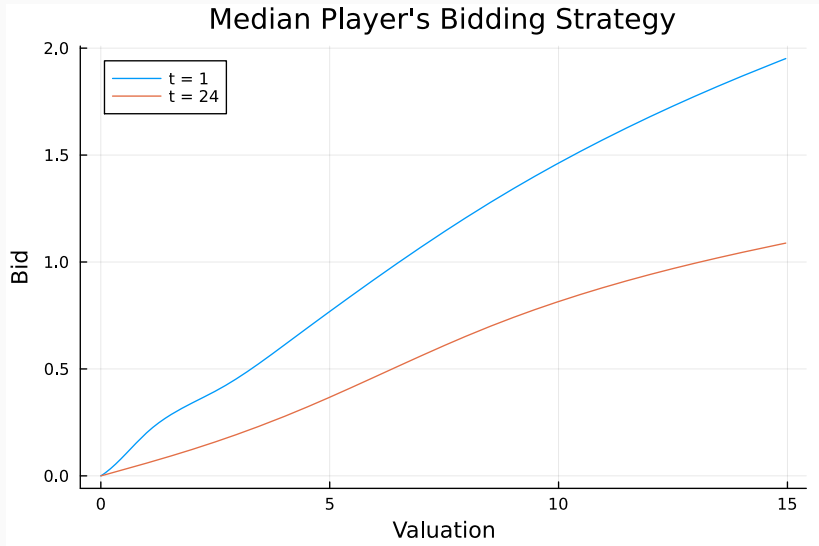
- $C \sim \text{TruncatedNormal}(\mu_C, \sigma_C)$
- $X \sim \text{LogNormal}(\mu_X, \sigma_X)$

◀ back

Distribution of daily remaining budget



Bidding Strategy ($t = 1$ vs $t = 24$)



Quantify impact of intertemporal budget constraints

- Avg. markdown $\frac{\text{valuation}-\text{bid}}{\text{valuation}}$ is 83%
- Simulate a bidder's behavior after removing its budget constraint
 - by setting $\eta = 0$
 - CF avg. markdown is 59%

- Decomposition of markdown:

$$83\% = 59\%(\text{static markdown}) + 24\%(\text{dynamic markdown})$$

- Similarly, dynamic incentives decrease entry rate by 25 percentage points from the CF counterpart

CF Basic Results

Auction Format	First Price	Second Price
Price Average	\$2.364	\$2.362
Price Variance	1.1246	3.565
Expected Total Revenue	\$480,427.33	\$480,073.49
Expected Total Bidder Surplus	\$1,191,000	\$1,185,000

[← back](#)

Intertemporal budget constraints lead to risk aversion

- Dynamic budget constraints lead bidders to exhibit risk aversion.
 - Concave continuation value: $EV_t(w_{it} - S_{it})$
- Echoes well-documented findings in finance that firms exhibit risk aversion when they have financial constraints (Froot et al., 1993; Opler et al., 1999).
 - Concave value function (Milne and Robertson, 1996; Holt, 2003; Rochet and Villeneuve, 2005).

References

- Aguirregabiria, Victor and Pedro Mira (2007) "Sequential Estimation of Dynamic Discrete Games," *Econometrica*, 75 (1), 1–53, 10.1111/j.1468-0262.2007.00731.x.
- Bajari, Patrick, C. Lanier Benkard, and Jonathan Levin (2007) "Estimating Dynamic Models of Imperfect Competition," *Econometrica*, 75 (5), 1331–1370, 10.1111/j.1468-0262.2007.00796.x.
- Bernhardt, Dan and David Scoones (1994) "A Note on Sequential Auctions," *The American Economic Review*, 84 (3), 653–657.
- Engelbrecht-Wiggans, Richard (1994) "Sequential Auctions of Stochastically Equivalent Objects," *Economics Letters*, 44 (1-2), 87–90, 10.1016/0165-1765(93)00327-K.

- Froot, Kenneth A., David S. Scharfstein, and Jeremy C. Stein (1993) "Risk Management: Coordinating Corporate Investment and Financing Policies," *The Journal of Finance*, 48 (5), 1629–1658, 10.1111/j.1540-6261.1993.tb05123.x.
- Gale, Ian L. and Donald B. Hausch (1994) "Bottom-Fishing and Declining Prices in Sequential Auctions," *Games and Economic Behavior*, 7 (3), 318–331, 10.1006/game.1994.1054.
- Holt, Richard W. P. (2003) "Investment and Dividends under Irreversibility and Financial Constraints," *Journal of Economic Dynamics and Control*, 27 (3), 467–502, 10.1016/S0165-1889(01)00057-4.
- Krishna, Vijay (2009) *Auction Theory*, Burlington, MA: Academic Press/Elsevier, 2nd edition.

- Li, Tong and Xiaoyong Zheng (2009) "Entry and Competition Effects in First-Price Auctions: Theory and Evidence from Procurement Auctions," *Review of Economic Studies*, 76 (4), 1397–1429, 10.1111/j.1467-937X.2009.00558.x.
- Milne, Alistair and Donald Robertson (1996) "Firm Behaviour under the Threat of Liquidation," *Journal of Economic Dynamics and Control*, 20 (8), 1427–1449, 10.1016/0165-1889(95)00906-X.
- Opler, Tim, Lee Pinkowitz, René Stulz, and Rohan Williamson (1999) "The Determinants and Implications of Corporate Cash Holdings," *Journal of Financial Economics*, 52 (1), 3–46, 10.1016/S0304-405X(99)00003-3.
- Rochet, Jean-Charles and Stéphane Villeneuve (2005) "Corporate Portfolio Management," *Annals of Finance*, 1 (3), 225–243, 10.1007/s10436-005-0018-7.

Weintraub, Gabriel Y., C. Lanier Benkard, and Benjamin Van Roy (2008)
“Markov Perfect Industry Dynamics With Many Firms,” *Econometrica*,
76 (6), 1375–1411, 10.3982/ECTA6158.